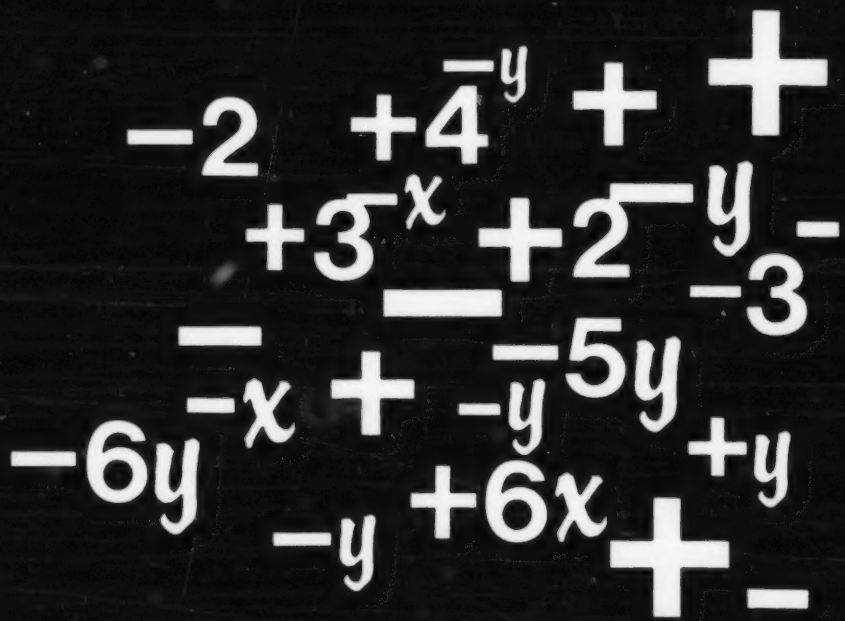
INTEGERS AND COMBINING LIKE TERMS IN ALGEBRA

1986



Manitoba Education
Curriculum Development and Implementation

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MANITOBA EDUCATION
CURRICULUM
DEVELOPMENT AND
IMPLEMENTATION

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The Algebra Methods Committee developed and implemented an inservice program which has lead to the writing of this module. They reviewed the contents of the module.

TABLE OF CONTENTS

Introduction.....vii
Unit Design.....1
Required Materials.....1
Colour Designations.....1
Blue and Red.....2
Writing Sentences.....4
The Opposite of a Blue or a Red.....6
Complete the Sentence.....8
Positive and Negative.....10
Integers.....12
Combining Integers.....14
Combine the Integers and Write a Sentence.....16
Combine the Integers.....18
Tantalizing Totals.....20
The Opposite.....22
Remove Parentheses and Combine Integers.....24
Multiplication of Integers.....26
Division of Integers.....28
Combine Like Terms.....30
Combine Like Terms and Write a Sentence.....32
Combining More Like Terms.....34
The Opposite of Algebraic Terms.....36
Remove Parentheses and Combine Like Terms.....38
Conclusion.....40

INTRODUCTION

The Algebra Methods Committee and the many teachers and administrators who participated in the Algebra Methods Inservices recommended to Manitoba Education that a descriptive module on the teaching of integers using concrete materials be developed for distribution to Manitoba mathematics teachers' grades 7 - 12.

Many students find operations with integers confusing. The double signs, misuse of language, and the abstract approach has contributed to this confusion. To correct this situation a concrete approach using bingo chips has been developed. The operations of addition and subtraction have been redefined as "combining". The language for combining integers has been reduced to positive, negative and the opposite of. The concrete approach and simplification of language in integers provides the required background for algebraic topics in the mathematics curriculum.

The aim of this unit is to assist teachers in the presentation of integers and the combining of like terms in algebra in a more interesting and meaningful way.

Senior high teachers should become familiar with the language and the processes of combining integers and algebraic terms as outlined in this module. The language and processes should be adapted to the teaching methods.

This is a module for teachers. It is recommended that teachers read and prepare the whole unit before teaching it.

UNIT DESIGN

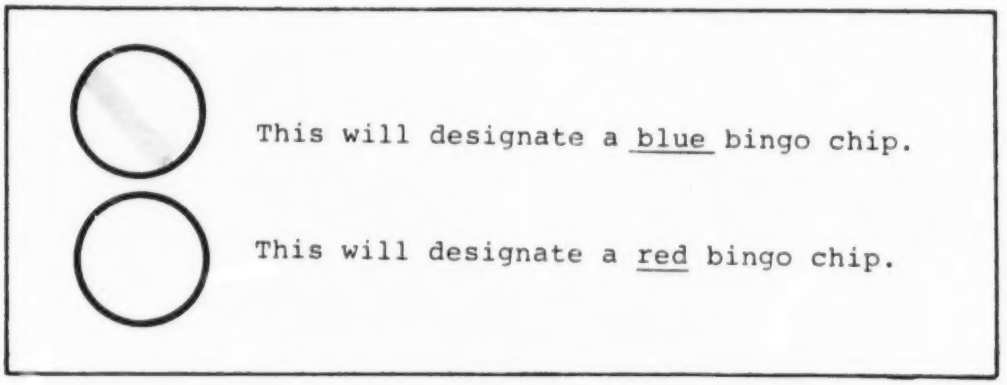
The pages on the left are the teaching instructions for the pages on the right which can be made into transparencies.

REQUIRED MATERIALS

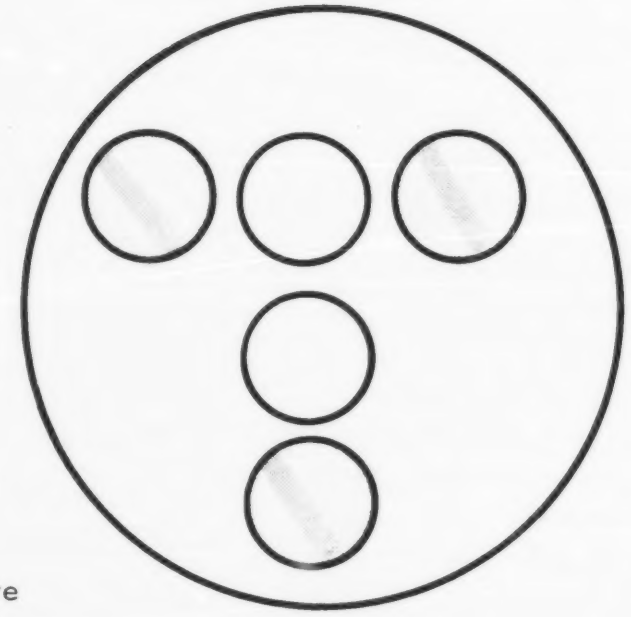
1. Twenty-five blue and 25 red bingo chips for teacher use and an equivalent number for every 3 or 4 students.
2. Permanent black marker(s).
3. Petri dish (optional).

COLOUR DIRECTIONS

This module can not be printed in two different colours due to prohibitive printing costs. Since most bingo chips that are stocked in stores are blue or red, the module is structured with blue and red colours in mind. The blue and red bingo chips will be represented by the designation below.



BLUE AND RED



Place 3 blue bingo chips and 2 red bingo chips in a petri dish as illustrated in the figure. Now place the petri dish with bingo chips on the overhead projector. Ask students the following questions:

Question: Of which colour are there more?

Answer: Blue

Question: How many more?

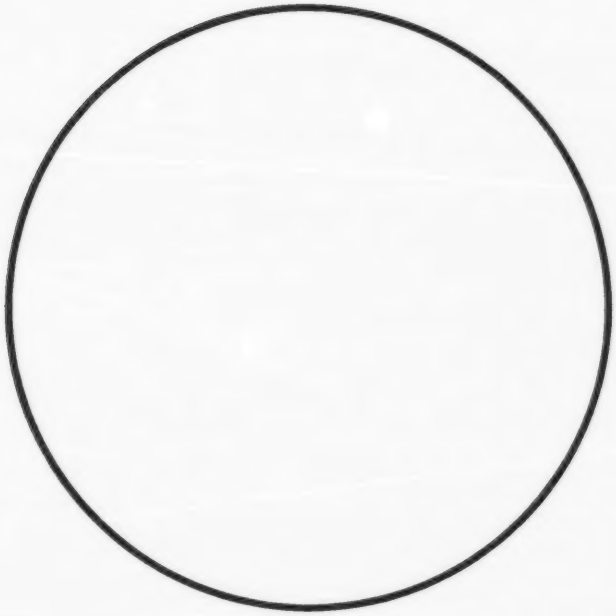
Answer: One

Repeat this procedure using a different number of blue and red bingo chips until the students respond readily.

BLUE AND RED

OF WHICH COLOUR ARE THERE MORE?

HOW MANY MORE?

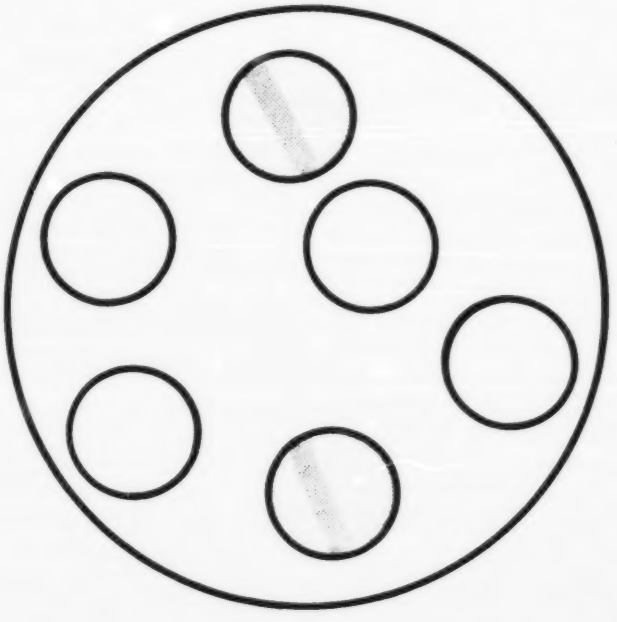


WRITING SENTENCES

Using similar examples as illustrated on the previous two pages, extend the procedure to write a sentence.

Example:

Place 2 blue bingo chips and 4 red bingo chips in a petri dish as illustrated in the figure. Now place the petri dish on the overhead projector. Ask the students the following questions.



1. How many are blue?
2. How many are red?
3. Of which colour are there more?
4. How many more?

As the students answer the above questions, write the sentence that describes the model on the overhead.

2 BLUE AND 4 RED = 2 RED

NOTE: USE AND AND NOT "+".

WRITING SENTENCES

WRITE A SENTENCE TO DESCRIBE THE FIGURE ABOVE.

1. _____ BLUE AND _____ RED = _____

2. _____ BLUE AND _____ = _____

3. _____ AND _____ RED = _____

4. _____ AND _____ = _____

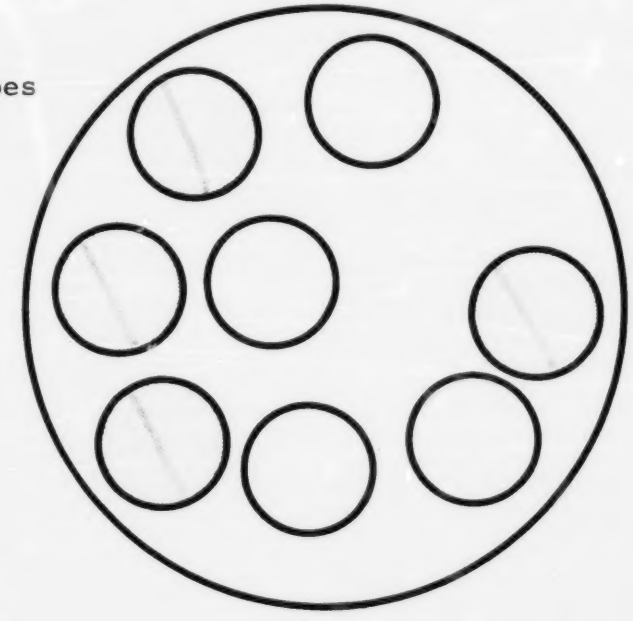
THE OPPOSITE OF A BLUE OR A RED

Give each student a copy of the page on the right.

Exercise 1: Read and illustrate with a model on the overhead projector.

Exercise 2: Read and illustrate with a model on the overhead projector. Move each red bingo chip next to a blue bingo chip as illustrated in the figure. This action will emphasize a red is the <u>opposite</u> of a blue.

Now write the sentence that describes the model.



4 BLUE AND 4 RED = 0

Have students do exercises 3 and 4.

THE OPPOSITE OF A BLUE OR A RED

1. PLACE ONE RED AND ONE BLUE ON THE PAPER.
   THIS IS A VALUE OF ZERO.
   THE SENTENCE WHICH DESCRIBES THIS RESULT IS 1 RED AND 1 BLUE = 0

2. PLACE FOUR RED AND FOUR BLUE ON THE PAPER.
   THE VALUE IS _____.
   THE SENTENCE THAT DESCRIBES THIS RESULT IS _____.
   SINCE ADDING THE SAME NUMBER OF RED AND BLUE RESULTS IN A SUM OF 0,
   THEN A RED IS THE OPPOSITE OF A BLUE.
   4 BLUE IS THE OPPOSITE OF 4 RED BECAUSE THE SUM IS 0.

3. PLACE FIVE BLUE ON THE PAPER WITH TWO RED.
   THE VALUE IS 3 BLUE.
   THE SENTENCE THAT DESCRIBES THIS IS 5 BLUE AND 2 RED = 3 BLUE
   THE ANSWER IS 3 BLUE BECAUSE THERE ARE 3 MORE BLUE.

4. PLACE 5 RED ON THE PAPER WITH 2 BLUE.
   THE VALUE IS _____.
   THE SENTENCE THAT DESCRIBES THIS IS _____.

COMPLETE THE SENTENCE

Distribute a copy of the page on the right to each student and have them complete the sentences or make a transparency and complete the sentences on the overhead projector with them. Be sure to emphasize the word AND.

COMPLETE THE SENTENCE

1. 4 BLUE AND 2 RED = _____

2. 3 BLUE AND 2 RED AND I BLUE = _____

3. 2 RED AND 4 BLUE AND 2 BLUE = _____

4. 5 BLUE AND 2 BLUE AND 5 RED = _____

5. I2 RED AND IO BLUE AND I RED = _____

6. 2 BLUE AND 5 RED AND 3 BLUE = _____

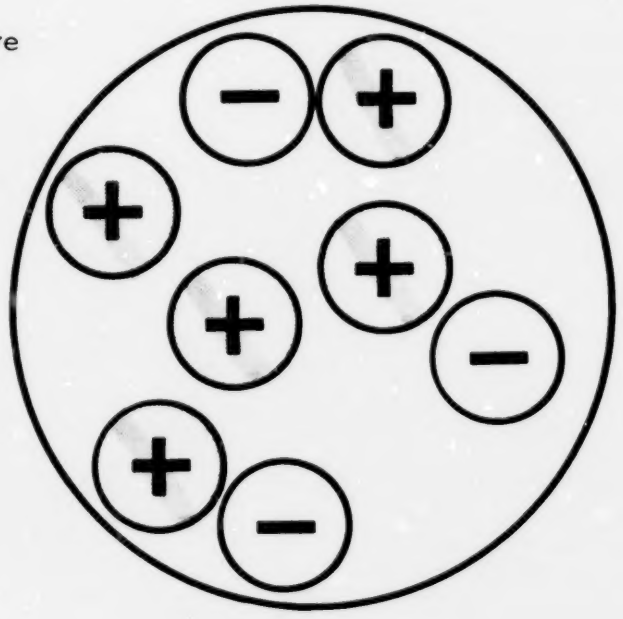
POSITIVE AND NEGATIVE

Using a black permanent overhead pen put a "+" on eight blue bingo chips and a "-" on eight red bingo chips as illustrated.

USE POSITIVE TO DESCRIBE THE SYMBOL +.

USE NEGATIVE TO DESCRIBE THE SYMBOL -.

Place 5 blue bingo chips with positive signs on them and 3 red bingo chips with negative signs on them in a petri dish. Place the petri dish on the overhead projector. Ask students the following questions.



Question: Are there more positives or more negatives?

Answer: Positives.

Question: How many more?

Answer: Two.

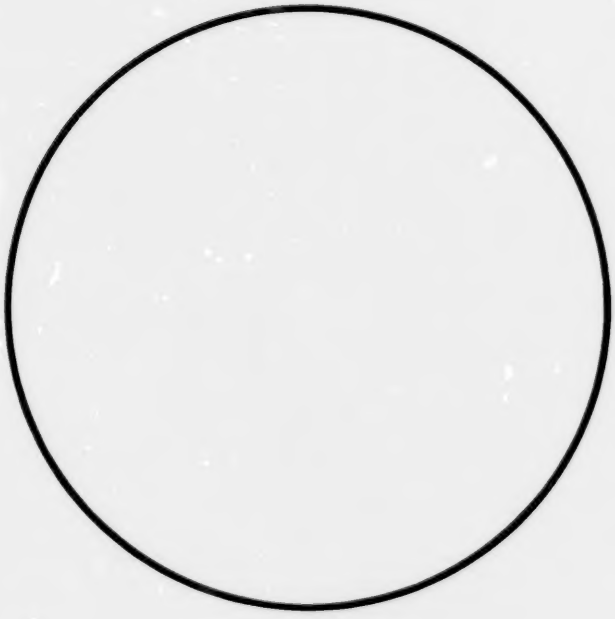
Now move each negative next to a positive as illustrated in the figure. This action will emphasize that a negative is the <u>opposite</u> of a positive.

Do several different examples until the students readily understand.

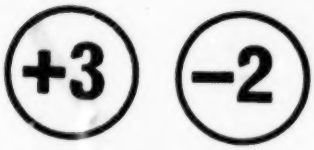
POSITIVE AND NEGATIVE

ARE THERE MORE POSITIVES OR MORE NEGATIVES?

HOW MANY MORE?

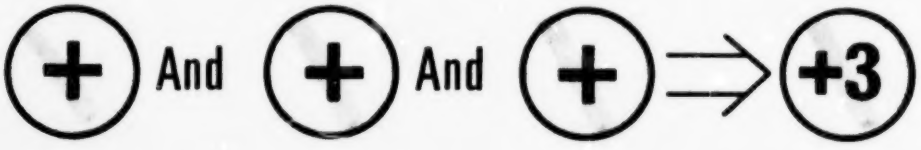


INTEGERS



Using a black permanent overhead pen put a "+3" on a blue bingo chip and a "-2" on a red bingo chip as illustrated. Prepare at least one blue bingo chip for each of the positive integers from +1 to +6. Similarly, prepare at least one red bingo chip for each of the negative integers from -1 to -6.

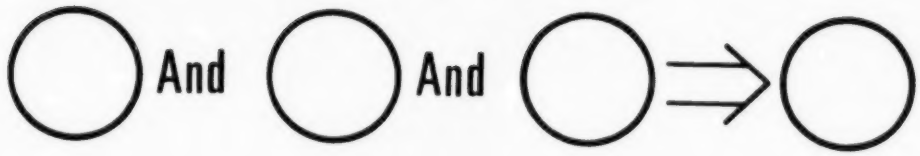
Using the page on the right illustrate that 3 positives will be represented by a single blue bingo chip with a +3 on it as shown below.



Illustrate that 2 negatives will be represented by a single red bingo chip with a -2 on it as shown below.

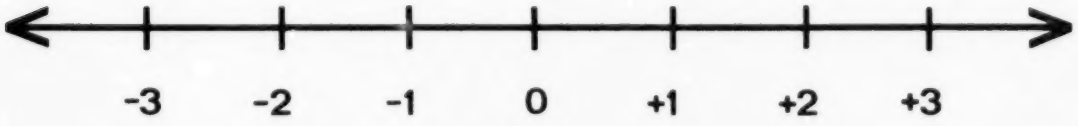
Explain integers, number line, positive integers and negative integers.

INTEGERS



$$\{\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots\}$$

THIS IS THE SET OF INTEGERS.



THIS IS CALLED A NUMBER LINE.

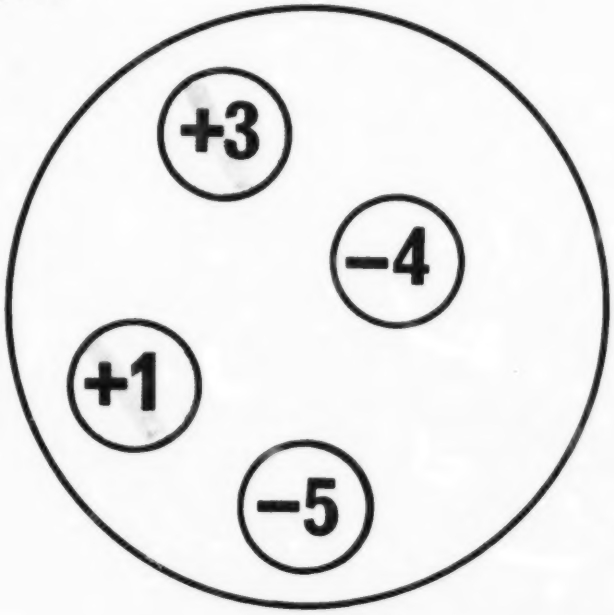
THE POSITIVE INTEGERS (OR NATURAL NUMBERS) ARE +1, +2, +3, ...

THE NEGATIVE INTEGERS ARE -1, -2, -3, ...

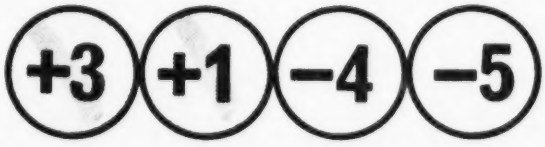
0 IS NEITHER POSITIVE NOR NEGATIVE.

COMBINING INTEGERS

Place 3 or more bingo chips with integers marked on them in the petri dish as illustrated in the figure on the right. Ask the students the questions. Are there more positives or more negatives? How many more?

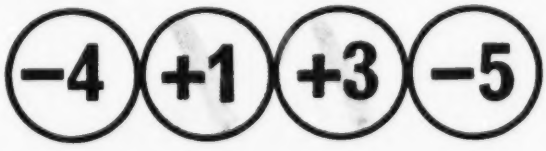


Now move the bingo chips to form a horizontal line as shown below.



A mathematical sentence that describes this combination of integers is: +3 + 1 - 4 - 5 = -5

Move the bingo chips to form a different horizontal arrangement as shown below.



A mathematical sentence that describes this combination of integers is:

- 4 + 1 + 3 - 5 = - 5

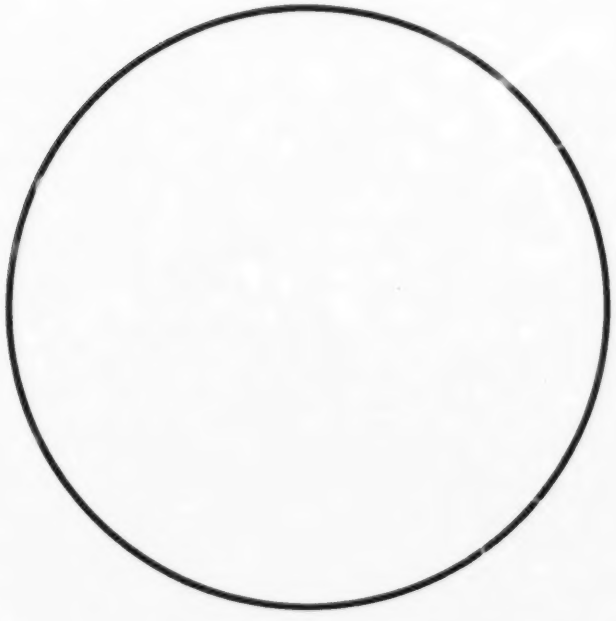
The rearranging of the bingo chips will show that the order of combining integers is irrelevant.

Do various examples until the students readily understand.

COMBINING INTEGERS

ARE THERE MORE POSITIVES OR MORE NEGATIVES?

HOW MANY MORE?



WRITE A SENTENCE THAT DESCRIBES THIS COMBINATION OF INTEGERS.

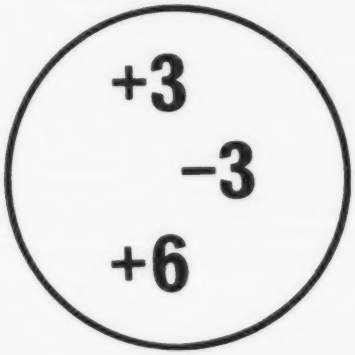
COMBINE THE INTEGERS AND WRITE A SENTENCE

Distribute a copy of the page on the right to each student. Have the students combine the integers and write a sentence.

If a student describes the combination of integers in the first figure as -3 +3 +6 = +6, then the student should read "negative three and positive three and positive six is equal to positive six".

COMBINE THE INTEGERS AND WRITE A SENTENCE

COMBINE THE INTEGERS IN EACH FIGURE AND WRITE A SENTENCE IN THE SPACE BELOW.



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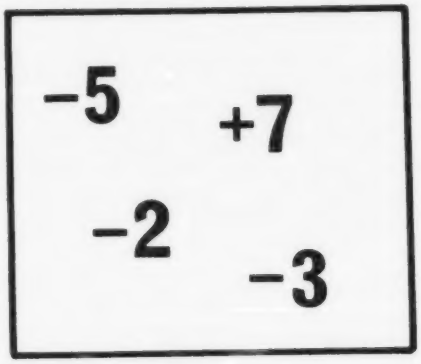

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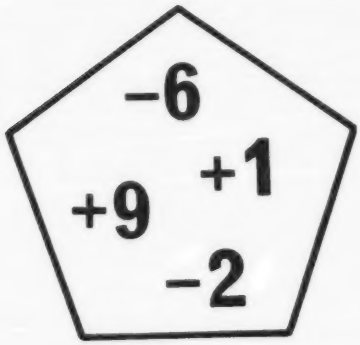

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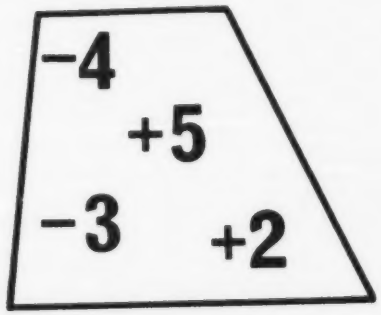

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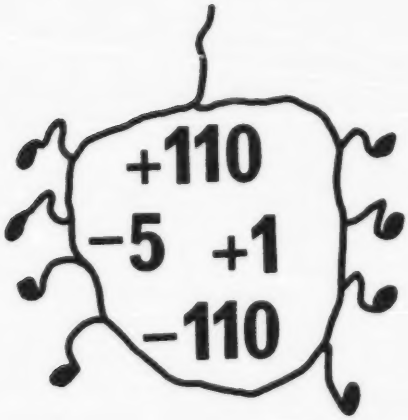

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COMBINE THE INTEGERS

Give the students time to complete the exercises on the right. Now have the students read aloud with you the question and answer to each. For example:

Exercise 1 is read " negative three and positive five and positive two is equal to positive four".

Exercise 2 is read "negative six and positive seven and positive six and negative five is equal to positive two".

Note: 1. It is essential that students master the correct reading of the sentences involving integers.

2. This is an exercise on combining integers and not the addition and subtraction of integers.

COMBINE THE INTEGERS

1.) $-3+5+2$

2.) $-6+7+6-5$

3.) $-5+7-2+3$

4.) $-2-1+5-6$

5.) $-1+1-1+1-1$

6.) $-50+60-50+40$

7.) $-64+32+38+64$

8.) $20-14+3-7-1$

TANTALIZING TOTALS

Tantalizing Totals is an exercise where students can make their own question. The exercise being self correcting, the students get immediate feedback. Have students do the page on the right.

MAGIC SQUARE

Have the students make a 3 X 3 magic square in which rows, columns and two diagonals have the same sum.

Rules: 1.) Use positive and negative integers.

2.) Use no integer more than twice.

Example:

+2	-5	+3
+1	0	-1
-3	+5	-2

This type of activity can be more productive than assigning many exercises on the combination of integers.

PROBLEM SOLVING STRATEGIES

Integer questions which require problem solving strategies should be assigned.

Example: Combine: $-2 + 3 - 4 + 5 - 6 + \ldots + 59 - 60$

Solution: $(1-2) + (3-4) + (5-6) + \ldots + (59-60)$

$= (-1) + (-1) + (-1) + \ldots + (-1)$

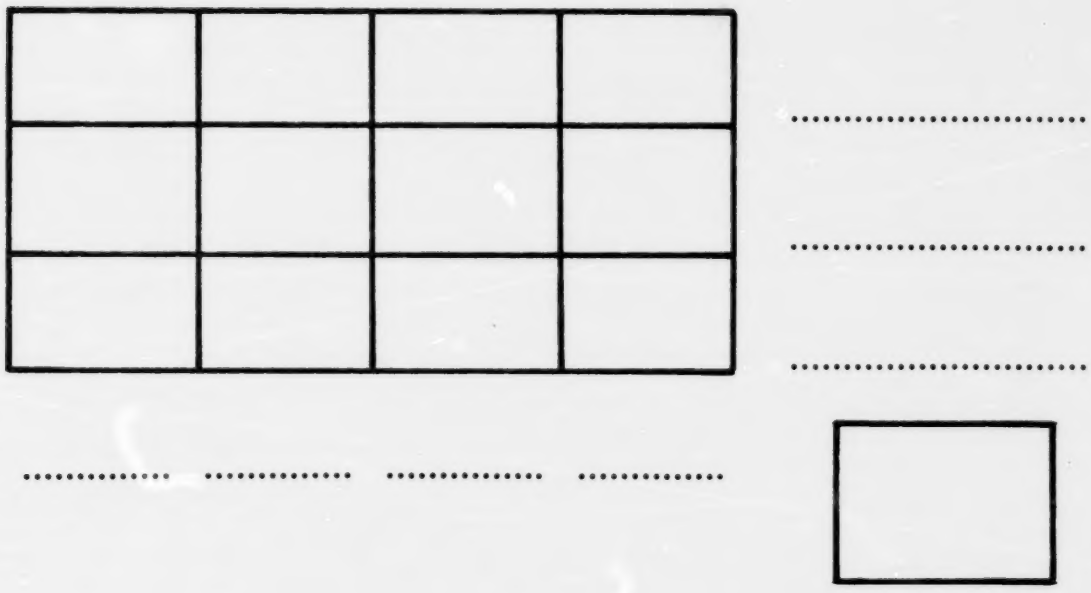
$= -30$

TANTALIZING TOTALS

1. PLACE A DIFFERENT INTEGER IN EACH SPACE, USING POSITIVE AND NEGATIVE INTEGERS.

2. COMBINE THE INTEGERS IN EACH ROW.

3. COMBINE THE INTEGERS IN EACH COLUMN.



IS THE ROW SUM EQUAL TO THE COLUMN SUM?

IF NOT, THERE IS AN ERROR.

THE OPPOSITE

Use the page on the right to explain that a negative before the parentheses means the opposite of the integer within the parentheses.

THE OPPOSITE

THE OPPOSITE OF 6 IS WRITTEN AS -6.

-6 IS THE OPPOSITE OF 6 BECAUSE -6 AND 6 EQUALS 0.

WRITE: $-6 + 6 = 0$

READ: "NEGATIVE SIX AND POSITIVE SIX IS EQUAL TO ZERO".

EXAMPLES:

A) $-(+3) = -3$

READ: THE OPPOSITE OF POSITIVE 3 IS EQUAL TO NEGATIVE 3.

B) $-(-5) = +5$

READ: THE OPPOSITE OF NEGATIVE 5 IS EQUAL TO POSITIVE 5.

C) $-(0) = 0$

READ: THE OPPOSITE OF ZERO IS EQUAL TO ZERO.

REMOVE PARENTHESES AND COMBINE INTEGERS

If an expression contains a negative before parentheses, take the opposite of the integer within parentheses and write the result. Then combine the integers.

Example: $+3 - (+2) - (-4)$

$= +3 -2 +4$

$= +5$

An expression with a positive before parentheses has the same meaning as if the positive before parentheses and the parentheses were not there.

For example $+ (-2)$ is equivalent to -2.

Example: $-3 +(-2) + (+6)$

$= -3 -2 +6$

$= +1$

When simplifying the expressions on the right the student should learn to read them correctly.

Example:

$-3 -(-2) +(-6)$ is read "negative 3 and the opposite of negative 2 and negative 6".

REMOVE PARENTHESES AND COMBINE INTEGERS

1) $+3 - (-2)$
   $= +3 +2$
   $= +5$

2) $-3 - (+5)$

3) $+5 - (+2)$

4) $-6 - (-3)$

5) $(-3) - (+2) - (-5)$

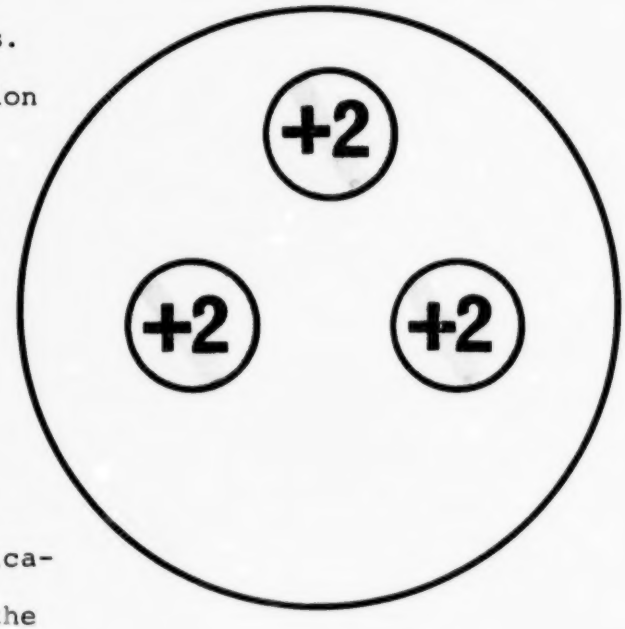
6) $(-2) + (-3) - (+4)$

7) $(-6) - (+3) + (-1)$

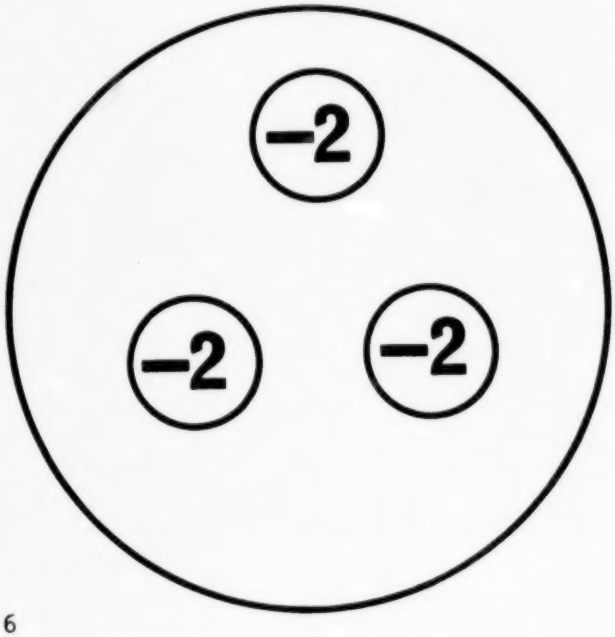
8) $-(-2) + (-3) - (-4)$

MULTIPLICATION OF INTEGERS

1. Product of two positive integers.
   Using the fact that multiplication is repeated addition, the model to the right demonstrates that;
   $(+2) + (+2) + (+2) = +6$
   or $(+3)(+2) = +6$.



2. Product of one positive integer and one negative integer.
   a) Using the fact that multiplication is repeated addition, the model to the right demonstrates that:
   $(-2) + (-2) + (-2) = -6$
   or $(+3)(-2) = -6$
   b) Multiplication is commutative, therefore:
   $(+3)(-2) = (-2)(+3) = -6$.

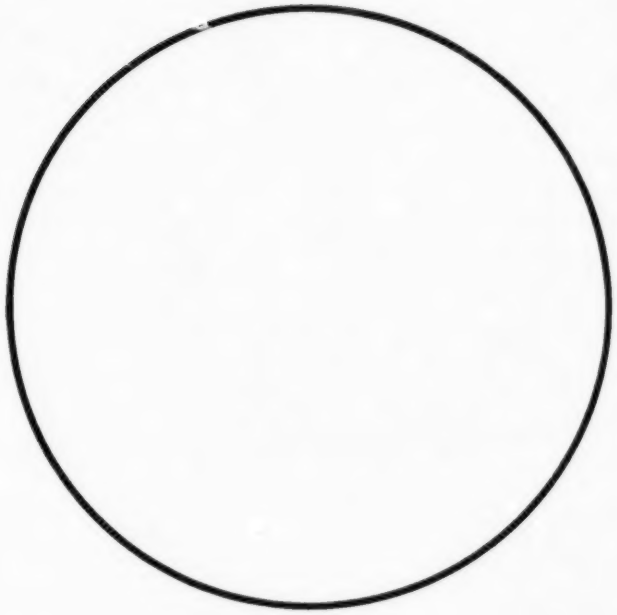


3. Product of two negative integers. Knowing that $-(+3) = -3$, it can be shown through logical reasoning that:

$$\begin{aligned} &(-3)(-2) \\ &= -(+3)(-2) \quad \text{since } -(+3) = -3 \\ &= -(-6) \quad \text{since } (+3)(-2) = -6 \\ &= +6 \end{aligned}$$

From these examples and others the students will understand the process of multiplication with integers.

MULTIPLICATION OF INTEGERS



1.) (+3)(+2)

2.) (A) (+3)(-2)

(B) (-2)(+3)

3.) (-3)(-2)

DIVISION OF INTEGERS

Use the fact that division is the inverse operation of multiplication to develop the process of division with integers. The process of multiplication with integers must be thoroughly understood before the process of division with integers can be developed.

Example:

Find $\frac{+\ 15}{-\ 3}$

The inverse of $\frac{+\ 15}{-\ 3} = \square$ is $\square\ (-3) = +\ 15$

Ask the students the following question:

What integer multiplied by -3 is +15?

Answer: - 5

Similarly use inverse operation and multiplication of integers to rationalize the processes of:

1) $\frac{+}{+}$ 2) $\frac{-}{-}$ 3) $\frac{-}{+}$

DIVISION OF INTEGERS

1) $\frac{+12}{+3} =$

2) $\frac{+20}{+5} =$

3) $\frac{-12}{-3} =$

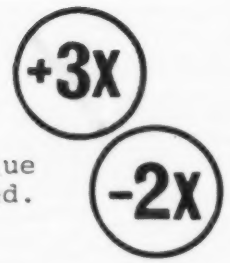
4) $\frac{-18}{-6} =$

5) $\frac{-12}{+3} =$

6) $\frac{-8}{+2} =$

7) $\frac{+12}{-3} =$

8) $\frac{+10}{-2}$



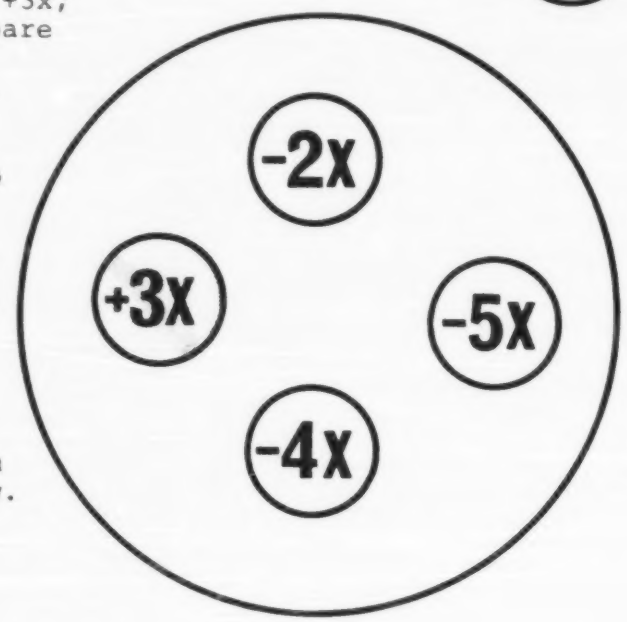
COMBINE LIKE TERMS

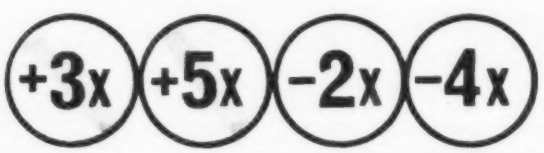
Using a black permanent overhead pen put a "+3x" on a blue bingo chip and a "-2x" on a red bingo chip as illustrated. Prepare at least one blue bingo chip for each of the following terms:+x, +2x, +3x, +4x, +5x and +6x. Likewise prepare at least one red bingo chip for each of the following terms: -x, -2x, -3x,-4x, -5x and -6x.

Place 3 or more bingo chips with positive and negative terms involving x in the circle on the transparency as illustrated to the right. Ask students the following questions.

Are there more positives or more negatives? How many more?

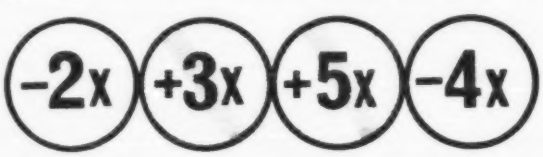
Now move the bingo chips to form a horizontal line as shown below.





A mathematical sentence that describes this combination of algebraic terms is: +3x +5x -2x -4x = +2x

Move the bingo chips to form a different arrangement as shown below.

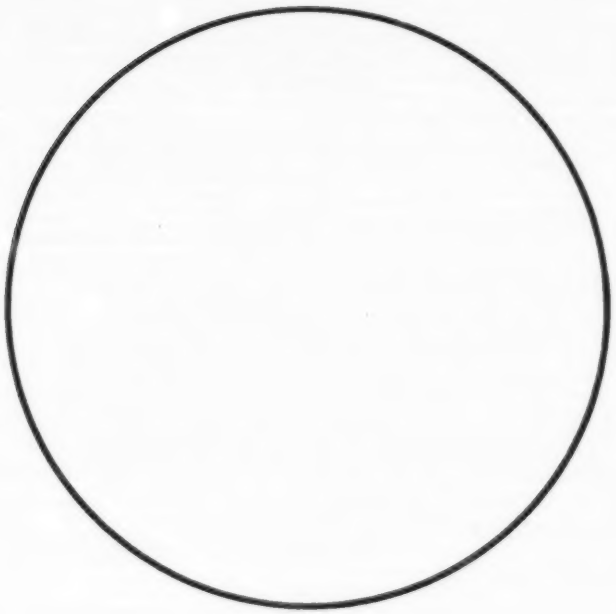


A mathematical sentence that describes this combination of algebraic terms is: -2x +3x +5x -4x = +2x

The rearranging of the bingo chips will show that the order of combining like algebraic terms is irrelevant. Do several examples until the students readily understand.

COMBINE LIKE TERMS

COMBINE THE TERMS AND WRITE A SENTENCE.



COMBINE LIKE TERMS AND WRITE A SENTENCE

Distribute a copy of the page on the right to each student and have them combine the like terms and write a sentence.

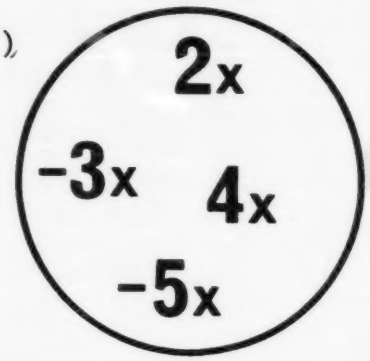
If a student wrote the like terms in question 1(A) as $2x + 4x - 3x - 5x = -2x$, then the mathematical sentence would be read as "positive 2x and positive 4x and negative 3x and negative 5x is equal to negative 2x".

Students should be able to read these algebraic expressions correctly.

COMBINE LIKE TERMS AND WRITE A SENTENCE

1. COMBINE LIKE TERMS IN EACH FIGURE AND WRITE A SENTENCE IN THE SPACE BELOW.

A.)



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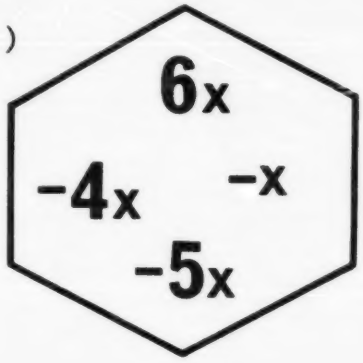
B.)



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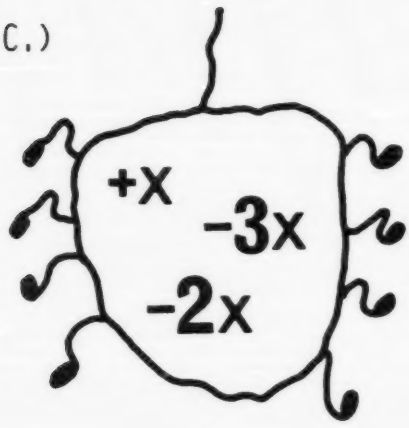
C.)



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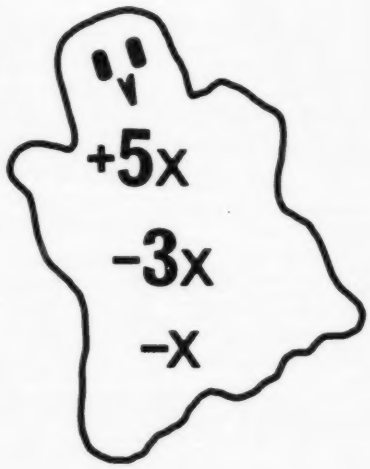
D.)



.....

2. COMBINE LIKE TERMS.

A) $7x - 9x + 6x$

B) $5x + x - 7x$

C) $-3x = 2x - 5x$

D) $x + 2x = 3x$

E) $7x + 3x - 5x + 2x$

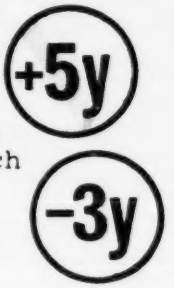
F) $-x - 7x - 4x + x$

G) $-2x + 3x - 5x + 2x$

H) $6x - x + 2x - 9x$

COMBINING MORE LIKE TERMS

Using a black permanent overhead pen put a "+5y" on a blue bingo chip and a "-3y" on a red bingo chip as illustrated. Prepare at least one blue bingo chip for each of the following terms: +y, +2y, +3y, +4y, +5y, and +6y. Similarly, prepare at least one red bingo chip for each of the following terms: -y, -2y, -3y, -4y, -5y and -6y. Also use the bingo chips with terms involving x which were prepared earlier. Place several bingo chips with positive and negative terms involving x and y in the circle on the transparency as illustrated to the right.
Ask students the following questions.

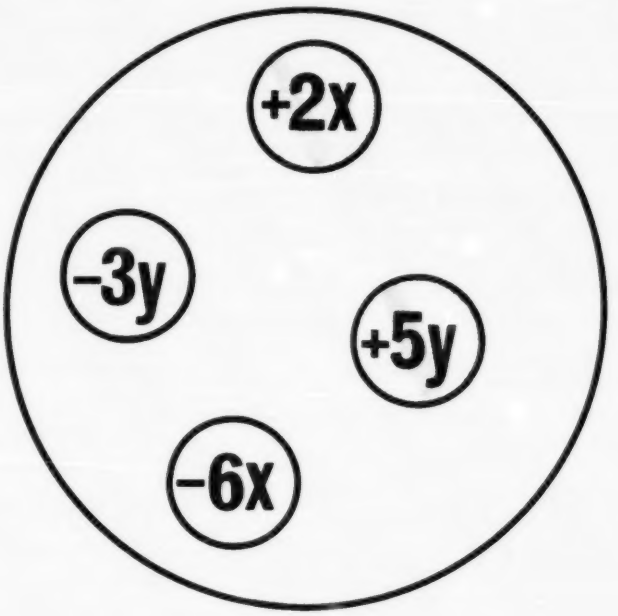


Are there more positives or more negatives involving x?

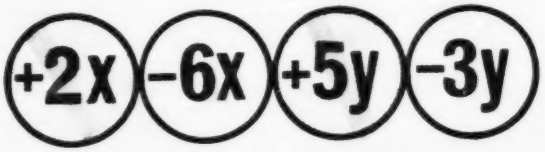
How many more?

Are there more positives or more negatives involving y?

How many more?



Now move the bingo chips to form a horizontal line as shown below.



A mathematical sentence that describes this combination of algebraic terms is: +2x -6x +5y - 3y = -4x +2y
This mathematical sentence would be read as "positive 2x and negative 6x and positive 5y and negative 3y is equal to negative 4x and positive 2x".

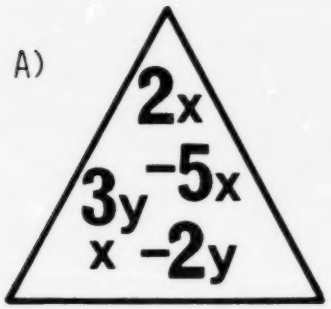
Rearrange the bingo chips and ask the students whether the answer will change. The rearrangement of the bingo chips will show that the order of combining like algebraic terms is irrelevant. Emphasize accurate reading of the sentences.

Do several examples until the students readily understand.

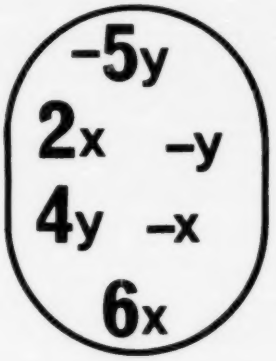
COMBINING MORE LIKE TERMS

I. COMBINE LIKE TERMS IN EACH FIGURE AND WRITE A SENTENCE IN THE SPACE BELOW.

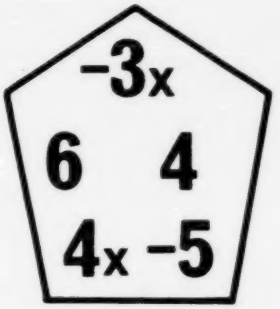
A)



B)



C)



....

2. COMBINE LIKE TERM.

A) 2X - 3Y - 6X - 2Y

B) -5X + 2Y - 3Y + 2X

C) 4Y - 2X + Y -3X

D) -2X + Y + 7X - 5Y

E) 10X - 3Y + X - Y - X

F) 9X +7Y - 3X - 2Y - Y

G) X + Y - Z - X + Y - 2Z

H) 2X - 3Y +Z + 3Y - 5Z

I) X - 2Y + 3X + Z - 5Y

J) -3X + 2 - 5X - 7

K) 9 - X + 2 - 7X + 5

L) 2X + Y - 7 - X + 3Y + 6

M) -5X + 6 - 7X + 4 + 9X - 10

N) 7X - 3 - 5X + 3 - 2X

O) 2X - 3Y + 5 - 2X + Y - 4

P) -XY +6XZ + 2XY - 4XZ

Q) 2XY - YZ + XY - 4YZ

R) -3XY + 2 - YZ - 5 + XY +YZ

S) 2XY - 3 + 2 - XY - X + 6

T) 3Y - 2 - XY + 5 - Y + 6XY

THE OPPOSITE OF ALGEBRAIC TERMS

Use the page on the right to explain that a negative before parentheses means the opposite of the sign of the algebraic term or terms within the parentheses.

THE OPPOSITE OF ALGEBRAIC TERMS

THE OPPOSITE OF X IS WRITTEN AS - X.

-X IS THE OPPOSITE OF X BECAUSE -X AND X EQUALS 0.

WRITE: $-X + X = 0$

READ: "NEGATIVE X AND POSITIVE X IS EQUAL TO ZERO".

EXAMPLES

A) $-(+2X) = -2X$

READ: THE OPPOSITE OF POSITIVE 2X IS EQUAL TO NEGATIVE 2X.

B) $-(-6X) = +6X$

READ: THE OPPOSITE OF NEGATIVE 6X IS EQUAL TO POSITIVE 6X.

C) $-(2X - 3) = -2X + 3$

READ: THE OPPOSITE OF POSITIVE 2X AND NEGATIVE 3 IS EQUAL TO NEGATIVE 2X AND POSITIVE 3.

REMOVE PARENTHESES AND COMBINE LIKE TERMS

If an expression has a negative before parentheses take the opposite of each term within parentheses and write the result. Then combine like terms.

Example: $-5x - (2x - 3)$

$= -5x - 2x + 3$

$= -7x + 3$

An expression with a positive before parentheses has the same meaning as if the positive before parentheses and the parentheses were not there. For example $2x + (3x - 5)$ is equal to $2x + 3x - 5$. Both are read "positive 2x and positive 3x and negative 5".

Example: $-3x + (2x - 6)$

$= -3x + 2x - 6$

$= -x - 6$

When simplifying the expressions on the right the students should learn to read them correctly.

Example: $-2x - (3x - 5) + (4x + 1)$ is read "negative 2x and the opposite of positive 3x and negative 5 and positive 4x and positive 1".

REMOVE PARENTHESES AND COMBINE LIKE TERMS

1) $-3X - (2X - 3)$

2) $-(2X - Y) + (Y - X)$

3) $-(-2X) - (X - 1) + (3 - X)$

4) $5 - [-2 - (X - 3) + (6 - X)]$

CONCLUSION

As stated in the introduction, this module has been developed to assist teachers in the presentation of integers and the combining of like terms in algebra in a more interesting and meaningful way. Teachers who use the concrete approach and the language development as outlined in this module should find that the students will combine integers and like terms in algebra with more ease and greater understanding.